




International Journal of Learning, Teaching and Educational Research
 Vol. 25, No. 2, pp. 547-576, February 2026
<https://doi.org/10.26803/ijlter.25.2.25>
 Received Nov 29, 2025; Revised Jan 27, 2026; Accepted Jan 31, 2026

Enhancing South African Mathematics Teachers' Big Ideas Knowledge and Self-Directed Learning: A blended Learning Professional Development Intervention

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Abstract. The South African school mathematics curricula does not emphasize the big ideas, these are only briefly mentioned and there is no elaboration on the benefits of teaching according to the big ideas. Additionally, professional development for school teachers in South Africa does not focus on teaching according to the big ideas in mathematics. Hence, the principal intention of this research was to ascertain how to enhance mathematics teachers' self-directed learning, by developing knowledge of big ideas in mathematics. Within a blended learning professional development programme, the study conceptualizes teachers' engagement with the big ideas in mathematics, utilizing self-directed learning as the guiding framework. This basic qualitative empirical study purposively selected all the Senior Phase and Further Education and Training mathematics teachers at a school (n = 5). Data was generated from an initial and concluding focus-group interview and activity submissions from a blended learning professional development programme. The study's findings suggested it was effective in developing the participants' self-directed learning characteristics, collaboration, lifelong learning and increased motivation. Simultaneously, the acquisition of big ideas knowledge resulted in improved curriculum continuity, initiated teachers to relook at the way that they teach mathematics to promote a more coherent and integrated learning experience, highlighted the real-world relevance and encouraged them to self-direct their learning in the future. Consequently,

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this study has the potential to close the identified gap in literature and to form the basis for designing and creating in-service professional development programmes and possibly even pre-service training programmes for mathematics teachers in the future.

Keywords: self-directed learning; mathematics big ideas; mathematics-teacher professional development; blended-learning intervention

1. Introduction

Linking mathematical concepts through big ideas is crucial, nevertheless, this approach is often overlooked. In the South African context, these big ideas are only briefly referenced in the national *Mathematics Teaching and Learning Framework* (Department of Basic Education [DBE], 2018), with little elaboration on their pedagogical significance or guidance on implementation. Askew's (2013) analysis of the Curriculum Assessment Policy Statement (CAPS) Foundation Phase (FP) and Intermediate Phase (IP), revealed a lack of explicit attention to the big ideas. This author asserts that the focus seems to be on developing learners "in preparation for more abstract mathematics in Higher/Tertiary Education institutions" (DBE, 2011, p.10) in the Further Education and Training (FET) Phase document. Mathematics teachers use the prescribed CAPS, which consists of isolated topics (Mahlaba, 2020; Olivier, 2016). Detrimentally, the cohesion between previously learnt work and the work to come is lost (Siemon, 2022).

Additionally, Olivier's (2016) reflection on in-service mathematics Professional Development (PD) programmes for secondary school teachers across several South African universities, revealed that none of the programmes focused on teaching according to the big ideas of mathematics. This consistent absence across institutions serves as strong evidence that PD for mathematics teachers in South Africa is lacking in this area, limiting teachers' capacity to be trained how to facilitate learners in viewing mathematics as a connected whole. Conversely, the focus is more on isolated topics (Olivier, 2016). The continued low achievement rates in mathematics in South Africa can perhaps be a consequence of the disjointed knowledge of how mathematics is disseminated (Askew, 2013; Theron, 2025).

Dall (2022) corroborated this claim reporting that in the 2019 Trends in International Mathematics and Science Study (TIMSS), out of the thirty-nine countries, the Grade 9 South African learners were ranked second last. While a focus on the mathematics big ideas is essential, realizing their impact in classroom practice is dependent on teachers' capacity to adapt continuously, reflect and expand their instructional knowledge. Thus, emphasizing the relevance of Self-Directed Learning (SDL). This study does not address the dwindling number of quality mathematics teachers in South Africa (Luneta, 2024), but focuses mostly on PD for in-service mathematics teachers. Rosli and Aliwee (2021) found that an effective PD programme enhances the quality of mathematics teaching and learning. It has the possibility to strengthen commitment and motivation and to fashion elevated self-efficacy and a positive outlook.

It is a popular conviction that teachers should be lifelong learners (Admiraal et al., 2016; Dhaliwal, 2015). This practice ensures that they remain relevant in an ever-changing world, when facilitating learning. As such, in many countries PD opportunities have been created to provide in-service teachers with more learning possibilities at school (Admiraal et al., 2016). Teachers' purpose for learning is to enrich their working environment and to aid them in achieving a specific goal. Sebotsa et al. (2021) argue that PD programmes should be underpinned by SDL. Knowles (1975) defined SDL as:

"A process by which individuals take the initiative, with or without the assistance of others, in diagnosing their learning needs, formulating learning goals, identifying human and material resources for learning, choosing and implementing appropriate learning strategies, and evaluating learning outcomes." (p.18)

For this study, the key elements featured in Knowles' (1975) SDL definition were taken into consideration.

Therefore, if teachers' SDL is effectively enhanced, they will be better equipped to learn about and facilitate learning according to the big ideas in mathematics. Furthermore, knowledge of the big ideas can impart pedagogical focus and guidance, thus contributing towards the efficacy of mathematics teachers in facilitating learning and therefore, resulting in better learner comprehension in the classroom (Mitchell et al., 2016; Tout & Spithill, 2015).

This study responds to this need by exploring a PD intervention designed to introduce and deepen teachers' understanding of big ideas in mathematics, while simultaneously enhancing their SDL capabilities. It emphasizes the significance of enhancing SDL in experienced South African private school mathematics teachers and that a Blended Learning (BL) PD programme has the potential to achieve this objective; a perspective earmarked previously in various research studies (Sriarunrasme et al., 2015; Uz & Uzun, 2018). Fadillah et al. (2020) believe that BL allows learners to guide their own learning process, dictate when they learn, how much they learn and whether they would require assistance from the facilitator. These aspects of BL would enable the development of SDL.

Hence, the principal intention of this research was to ascertain how to enhance mathematics teachers' SDL, while developing knowledge of big ideas in mathematics through a BL PD programme. The subsequent research objectives were compiled to answer the research questions:

- To establish what the body of scholarship reveals about SDL and PD in mathematics, particularly with regard to big ideas in mathematics.
- To determine what perceptions a South African private school's mathematics teachers, have with regard to SDL skills.
- To ascertain a South African private school's mathematics teachers' knowledge of the big ideas in mathematics.
- To clarify how a South African private school's mathematics teachers experience a BL PD programme on big ideas in mathematics.

This study therefore offers a novel contribution in several respects, suggesting that the big idea in mathematics is worthwhile content to include in a mathematics BL PD programme. Moreover, this study highlights the importance of teachers acquiring knowledge of the big ideas in mathematics. Especially, towards curriculum continuity and presenting a more interconnected perspective of mathematics to learners, no previous South African PD programmes have covered this. These findings provide empirical evidence that has the potential to close the gap identified in the literature and to form the basis for designing and creating in-service PD programmes and possibly even pre-service training programmes for South African mathematics teachers in the future.

This article begins with a review of the literature on BL as a context for professional growth, cultivating SDL and understanding and teaching big ideas in mathematics. Next, the methodology of study is described. Then, the results and findings are presented and analyzed, and finally, a discussion unfolds about the implications for enhancing mathematics teachers' SDL and acquiring knowledge of mathematics big ideas.

2. Literature Review

This study drew on theoretical perspectives related to SDL, BL and the role of big ideas in mathematics education to examine how a PD programme can support teacher learning in a conceptually coherent and autonomous manner.

2.1 Blended Learning as a Context for Professional Growth

As mentioned previously, BL environments are believed by many previous studies to enhance the development of SDL. In this study, BL was implemented through a Flex model in which teachers engaged in a combination of asynchronous online learning, face-to-face collaborative sessions and individualized instructional facilitation, with control over pacing, sequencing and engagement pathways. This study adopted the Flex model of BL (Staker & Horn, 2012), allowing learners to move between learning approaches according to an individualized, adaptable schedule. The instructor is available when required. Activities include intimate group instruction, group tasks and one-on-one tutoring. The Flex model was chosen specifically for its alignment with teachers' needs for individualized, adaptable schedules.

Theoretically, the autonomy inherent in BL aligns closely with the principles of SDL. Teachers rate the most effective PD models as those which include BL features. This is due to the opportune and flexible nature of this model in terms of pace, location and time (Hood, 2022). Practically, this type of model allows teachers to fulfil their professional responsibilities, and the structured support offers targeted assistance and collaboration with peers. Moreover, a noteworthy parallel exists between heightened SDL and BL models for PD (Wang et al., 2025), reiterating previous studies above.

Similarly, SDL involves agency in identifying learning needs, selecting resources and regulating one's learning trajectory (Robinson & Persky, 2020). When designed intentionally, a BL environment can therefore serve not only as a

delivery mechanism but as a scaffold for cultivating SDL in teachers. Even though several studies report positive connections between BL and the development of SDL, many concentrate on learners rather than in-service teachers and hardly any observe how autonomy is constructed gradually in PD programmes. This insinuates a need for research that investigates how BL models deliberately cultivate teachers' SDL.

2.2 Cultivating Self-Directed Learning

As mentioned, Knowles (1975) defines SDL as a process in which individuals take initiative in diagnosing learning needs, setting goals, identifying resources, implementing strategies and evaluating outcomes. Francom's (2010) review of various models of research, theory, teaching and learning revealed that the development of SDL skills is reliant on the alignment of learner readiness with the demands of SDL, enabling learners to assume responsibility for their learning gradually. The simultaneous acquisition of SDL and subject-matter knowledge should be supported, aiding learners to incorporate both effectively. Learning tasks replicating professional contexts should be created to practise SDL, in so doing connecting theoretical understanding and practical application.

All four of Francom's (2010) principles informed the design of this study's BL PD programme, which positioned teachers as both learners and professionals and aimed to develop their conceptual and reflective capacities simultaneously. Wittmann and Olivier (2019) propose that for South African teachers to overcome PD challenges, SDL must be fostered to enable teachers to perform a less passive role in their PD. Both the BL Flex model (Staker & Horn, 2012) chosen for the PD programme and the suggested principles (Francom, 2010) that should be used to cultivate SDL apply to assisting teachers with the acquisition of knowledge of, and experience with, the big ideas in mathematics.

2.3 Understanding and Teaching Big Ideas in Mathematics

The term *big ideas* in mathematics refers to central, unifying concepts that structure mathematical knowledge and give coherence to the curriculum (Steen, 1990; Tout & Spithill, 2015). These ideas provide a framework that supports the progression and integration of mathematical understanding across grades. In various international contexts, such as the Australian Curriculum and PISA's mathematical literacy framework, big ideas serve as anchors for both content organisation and pedagogical decision-making (Mitchell et al., 2016; Tout & Spithill, 2015). Mitchell et al. (2016) explored a conceptual perspective, dealing with big ideas in terms of content, learning and the type of domain. The consensus though, is that big ideas create an integrated view of mathematics and its uses in the real world (Tout & Spithill, 2015).

Hypothetically, South African teachers' capacity to self-directed their learning is mediated by their interest in, and perceived relevance of, the mathematical knowledge encountered in PD. When teachers are able to visualize the positive effect that learning about the big ideas in mathematics will have on their facilitation of learning, intrinsic motivation to self-direct learning is more likely to occur. Bishara (2021) declares that teachers require the development of SDL to achieve broadmindedness. This would contribute to creating an environment that

nurtures mathematics. Likewise, Nurnberger-Haag et al. (2024) reinforce the widely advocated significance of concentrating on the big ideas in mathematics teaching and learning. Even though mathematics big ideas are extensively identified as central to curriculum coherence, rather than focusing on teacher learning and PD, the body of scholarship is inclined to place emphasis on curriculum design. As a result, there is a gap in literature about how teachers themselves acquire conceptual coherence around the big ideas.

An integrated conceptual framework for this study is formed by combining BL, SDL and big ideas in mathematics. The Flex model of BL affords the structural setting for autonomy and adaptable engagement, which are essential for the advancement of SDL. Consequently, meaningful and reflective engagement with the mathematics big ideas, as opposed to a procedural approach, is enabled through SDL. Teachers' SDL is given direction and coherence because of the focus on big ideas, thus ensuring that autonomy, which assists deep conceptual comprehension, is employed. As a result of this interplay, teachers, as autonomous professionals, will actively construct coherent mathematical knowledge through a supportive BL PD programme.

The Mapungubwe Institute for Strategic Reflection (2022) identified that the South African mathematics teachers are challenged with a bloated curriculum, time constraints, limited in-service training and low efficacy in teaching mathematics due to poor learner results. Similarly, the body of scholarship presents mathematics teacher PD as a fragmented picture that focuses on general teacher development and isn't accessible to all teacher in the country (Mulaudzi & Mosia, 2025). This indicates the need for BL PD programmes that are flexible and encourage autonomy and concept integration.

3. Methodology

3.1 Design of the Study

This study was grounded in an interpretivist paradigm. Interpretivism is constructed on the belief that the researcher should become immersed in the study and interact closely with the participants to understand their lived interpretations (Mack, 2010). The first author, herself as a practicing mathematics teacher in the same school as the participants, brought an emic perspective to the research process, drawing on both prior academic work and daily experience with BL, SDL and mathematics teaching. The significance of the incorporation of the researcher's subjectivity, in gaining insight and understanding of the participants' views, was previously earmarked (Taylor & Medina, 2013). This study was hence framed through this theory. Due to the emic position of the researcher, an integral part of the research process was reflexivity; memo writing and reflective journaling throughout the collection and analysis of data were carried out to enable assumptions to surface, possible bias to be monitored and interpretations to be critically interrogated (Taylor & Medina, 2013).

Considering that an interpretivist paradigm underpinned this research, it followed that a qualitative research approach was the most suitable choice, enabling an in-depth exploration of teachers' subjective experiences within a BL

PD context. A qualitative research approach concentrates on gaining insight, discovering and understanding how the participants involved in a study perceive and make sense of their experiences and how they construct their everyday lives (Kivunja & Kuyini, 2017; Merriam & Tisdell, 2016). Qualitative studies are also most applicable for studies endeavouring to explain why a concept is, or is not, followed (Busetto et al., 2020). This research followed the most common form of qualitative education research; basic qualitative studies and therefore no strategy of inquiry was declared (Merriam & Tisdell, 2016). Kahlke (2014) asserts that when the strengths of one or a few of the established strategies of inquiry are used to construct the research design, the research can be considered a basic qualitative study.

The meaning in this study was *constructed* as the participants engaged with the BL PD programme. The private school mathematics teachers shared how they experienced the BL PD programme, which aimed to simultaneously enhance their SDL and develop knowledge of big ideas in mathematics. According to Merriam and Tisdell (2016), these characteristics are the foundation of constructivism. Similarly, the researcher was *interested in*, and wanted to *understand*, how the private school mathematics teachers interpret, construct and attribute meaning to experiencing a BL PD programme created to enhance SDL and develop knowledge of big ideas in mathematics. Thus, the researcher's intention coincided with Merriam and Tisdell's (2016) description of a basic qualitative study.

3.2 Context and Study Participants

The study drew on a purposive sample of five mathematics teachers from a private school in Johannesburg, South Africa. This school was selected for convenience, as the researcher and participants all worked at the same site, enabling ease of access and fostering rapport (Nieuwenhuis, 2019b). Although small, this was fitting for the nature of the study that sought to gain a comprehensive understanding of the individual teachers' experience and interpretation of the BL PD programme rather than to make a generalization, interpretivist and qualitative. A deep and thick dataset was collected because each participant contributed for an extended time, in the two focus-group interviews, submitted written and audio reflections and collaborative activities. This repeated interaction and extended engagement is suitable for small sample sizes according to Guest et al. (2020).

However, the sample size aligns with Krueger and Casey's (2015) guideline for focus group research, which recommends between four and twelve participants. Participants were selected based on inclusion criteria relevant to the study's aims. All taught mathematics in the Senior Phase (SP) and/or Further Education and Training (FET) Phase and followed the Independent Examinations Board (IEB) Subject Assessment Guidelines (SAGs), allowing for flexibility beyond the constraints of the national CAPS curriculum. Teachers were also required to have regular access to smart devices and stable internet connectivity to participate fully in the BL PD programme.

The sample comprised a diverse group of experienced teachers, including one teaching Grade 7, one teaching Grades 8-9, two teaching across SP and FET (including one trained in Zimbabwe) and one serving as head of the mathematics department. All had recent experience teaching online during the COVID-19 pandemic and were familiar with BL approaches in their classrooms. The school's commitment to professional development and curriculum continuity further supported the selection of this site.

This sample was not intended to be representative but to provide varied perspectives within a single teaching context, in line with the goals of a basic qualitative study grounded in an interpretivist paradigm (Merriam & Tisdell, 2016). The diversity of roles and backgrounds among participants enriched the understanding of how the PD programme influenced their self-directed learning (SDL) and engagement with big ideas in mathematics. Data saturation was reached within this group and therefore the sample was not expanded (Guest et al., 2020; Nieuwenhuis, 2019a).

3.3 Data Collection and Research Instruments

Qualitative data was generated through a combination of face-to-face engagement during the pre- and post-PD focus-group interviews, individual written and voice-note reflections and collaborative group activities within a BL PD programme. Semi-structured interviews, comprising open-ended questions and neutral prompts enabled the participants' perspectives without the researcher confirming or leading. These interviews documented participants' teaching experience, SDL skills and knowledge of big ideas.

The month-long BL PD programme, hosted on Google Classroom, consisted of a sequence of structured asynchronous and synchronous, individual and group activities. This online intervention facilitated flexible and self-paced learning. The written reflections on SDL (Bosch et al., 2019) and big ideas (Charles, 2005; Tout, 2015) enabled participants to evaluate and articulate their learning. Voice-note reflections promoted a salient element of SDL, critical self-reflection (Thornton, 2010; Song, 2021). Reflection is often associated with SDL (Song, 2021) and more specifically, Gencel and Saracaloglu (2018) claim that for SDL to occur, reflection is a prerequisite. Lee and Mori (2021) emphatically conclude that SDL is promoted through the reflective practices involved in collaboration, self-reflection and peer feedback. Collaborative group activities using shared Google documents fostered peer-supported learning and linked curriculum planning to big ideas (Bosch, 2017; Brandt, 2020).

The instruments' validity was ensured through triangulation across several sources, placement of activities with research constructs and the use of video recordings of both focus-group interviews to produce both oral and observation data. After each focus-group interview, verbatim transcriptions were compiled (Nieuwenhuis, 2019a). Taking into consideration the professional relationship of the participants and the researcher, the limitations of the self-reported data from the written and audio reflections, in terms of social desirability bias are acknowledged. The individual and asynchronous completion of the reflections, mitigated this, granting the participants' privacy and time. Furthermore, the

group dynamics, dominant influential voices, could have affected focus-group interviews. This was mitigated with the use of a semi-structured interview guide and the researcher's purposeful facilitation to ensure that each participant was equally heard.

3.4 Data Analysis

Data collected from video recordings, voice notes and written submissions were analyzed according to Braun and Clarke's (2006) six-phase process using thematic analysis. The thematic analysis allowed the researcher to induce relationships between concepts and measure their relative frequency in relation to the data as a whole (Jnanathapaswi, 2021). ATLAS.ti™ 22 supported systematic coding, theme development and visualization of relationships. To augment transparency in interpretation, trustworthiness and to differentiate the participants' meanings from researcher's knowledge of BL, SDL and mathematics teaching, reflexive memos were utilized.

A description of the six-phase process ensues:

- Phase 1 - Transcribe and scrutinize the generated data

The interviews and reflective activities were transcribed using HappyScribe. Although this automated transcription was used for efficiency, all transcripts were thoroughly checked for accuracy, omissions and spelling errors in line with Braun and Clarke's (2006) guidelines. The researcher engaged in memo writing and reflection throughout the process to support interpretive depth (DeJonckheere and Vaughn, 2019), while reviewing the videos and listening to the voice notes allowed immersion in the data and helped identify verbal and non-verbal dynamics (McMullin, 2023).

- Phase 2 - Initiate the coding process

Transcripts from the focus-group interviews and all the PD programme activity submissions, including the reflections, were uploaded to ATLAS.ti™ 22. To structure, categorise and thematically organise the collected data according to the study's sub-questions, a systematic coding process was employed (Williams & Moser, 2019). The coding process followed an inductive, data-driven approach used to generate codes reflecting participants' responses, enabling patterns and meaningful relationships to emerge across the dataset (Jnanathapaswi, 2021). The full set of emergent codes is shown in Table 1.

Table 1: Research codes in alphabetical order

Big ideas: after PD	Challenges: topics
Big ideas: after reading	Expectations for participating
Big ideas: benefits	Mathematics teaching experience
Big ideas: personal	SDL: aspirations
Big ideas: pre-knowledge	SDL: developed
Big ideas: school	SDL: currently possesses
Challenges: COVID	SDL: personal
Challenges: learners	SDL: pre-knowledge
Challenges: teachers	SDL: school

- Phase 3 – Search for themes

Codes generated in Phase 2 were examined for patterns and grouped into higher-order themes (Braun & Clarke, 2006). In ATLAS.ti™ 22, these themes, or “categories” (Friese, 2022), were organized hierarchically, with related codes assigned as subcodes (Braun & Clarke, 2006; Friese, 2022; Nowell et al., 2017). Not all codes aligned neatly with broader categories. As recommended in the literature, such standalone codes were not discarded but retained as independent categories, such as mathematics teaching experience and expectations for participation. This structured approach supported a clearer interpretation of participants’ perspectives. The complete list of initial codes is presented in Table 1 and Figure 1 displays the categories (with an icon) and their subcodes.

- Phase 4 – Review themes

The researcher reviewed and refined the themes identified in Phase 3 to ensure they accurately represented the data (Braun & Clarke, 2006). Subcodes were examined for completeness, with some merged to form broader, more coherent categories – for example, “expectations for participating” was merged with “SDL: aspirations”, and “mathematics teaching experience” with “SDL: currently possess”. Other related subcodes, such as those under “Big ideas” and “Challenges”, were combined to simplify the thematic structure. The resulting thematic map provided a clear overview of the themes and their interrelationships, reflecting the story told by the data (See Figure 2).

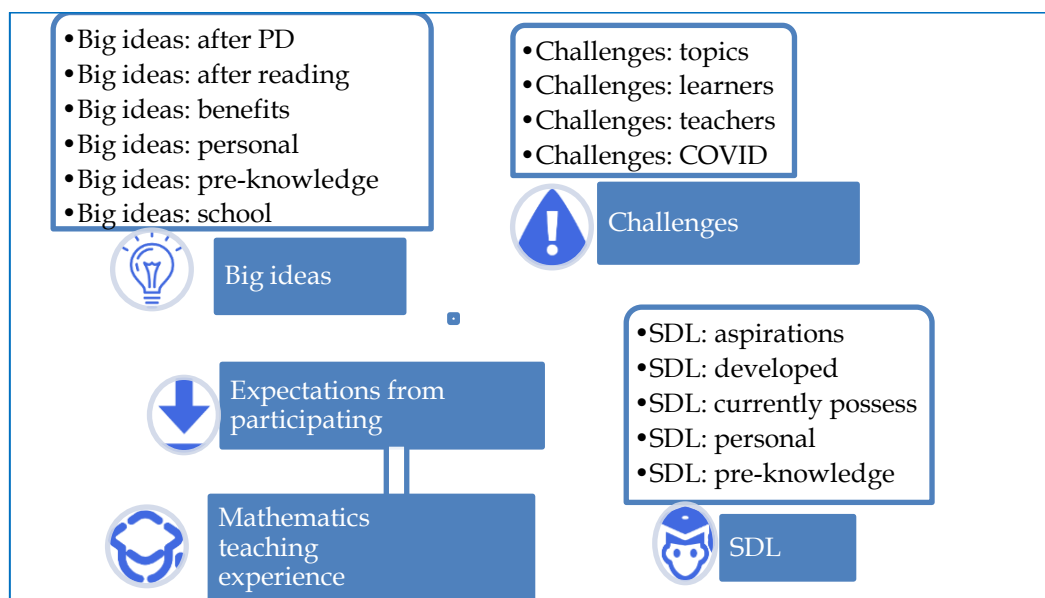


Figure 1: Research categories and their subcodes

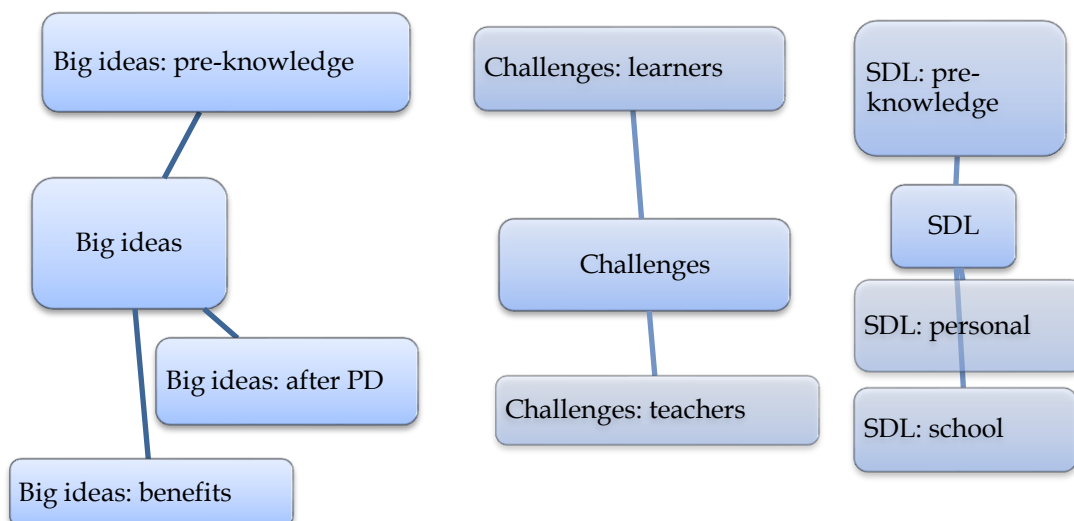


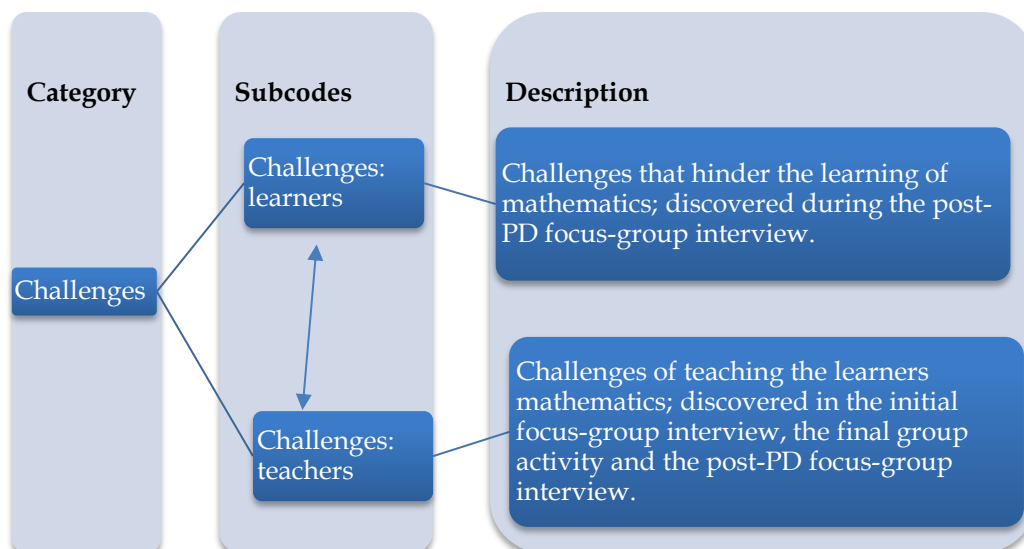
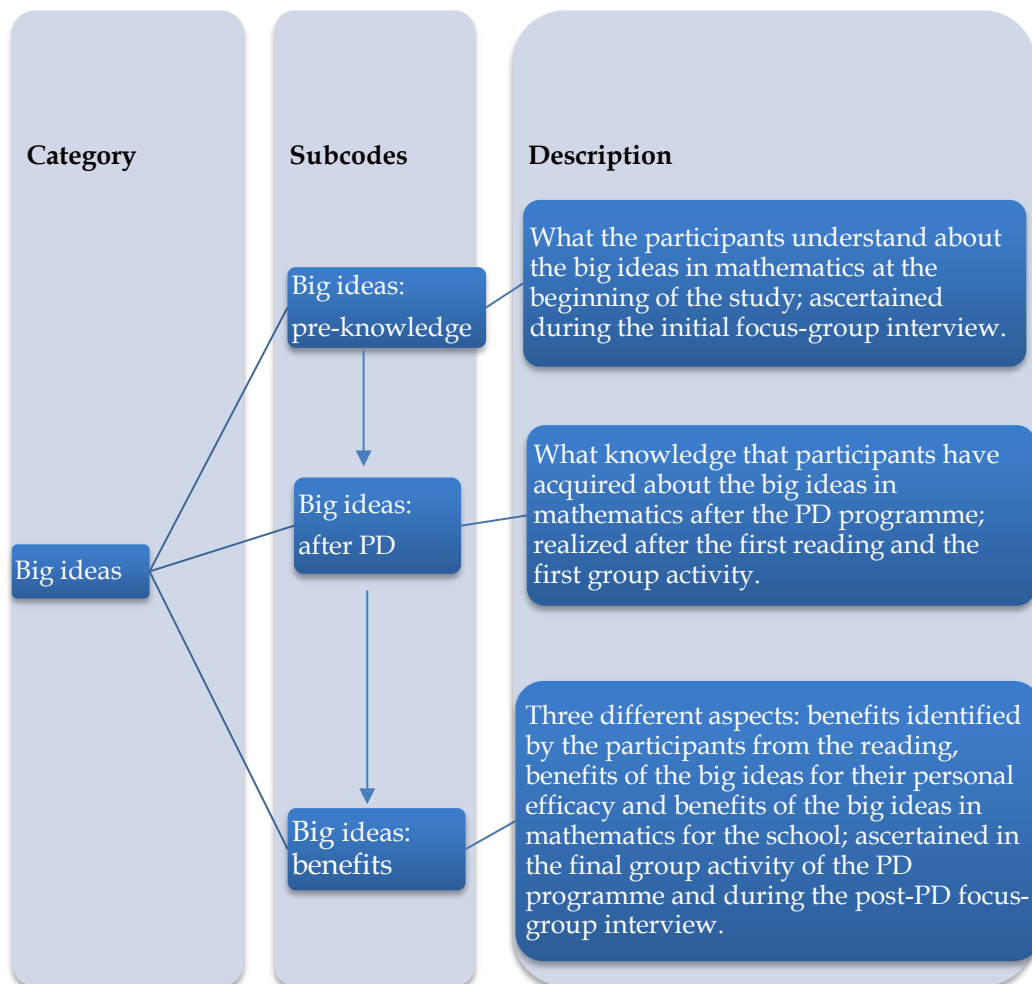
Figure 2: Final thematic map of the analysis

- Phase 5 - Define and name themes

Each theme was defined by its subcodes to capture the essence of what it represented in the data (Braun & Clarke, 2006). Clear definitions were developed for all categories (Figure 3), ensuring that each theme reflected a distinct aspect of participants' experiences and prepared the researcher for the final analysis and reporting (Nieuwenhuis, 2019b).

- Phase 6 - Produce the report of findings

The final phase consisted of the researcher writing a narrative report. The report used the definitions and themes to express the analysis findings and interpreted the subsequent findings of the thematic analysis, with particular reference to the study's research question (Bosch, 2017; Braun & Clarke, 2006).



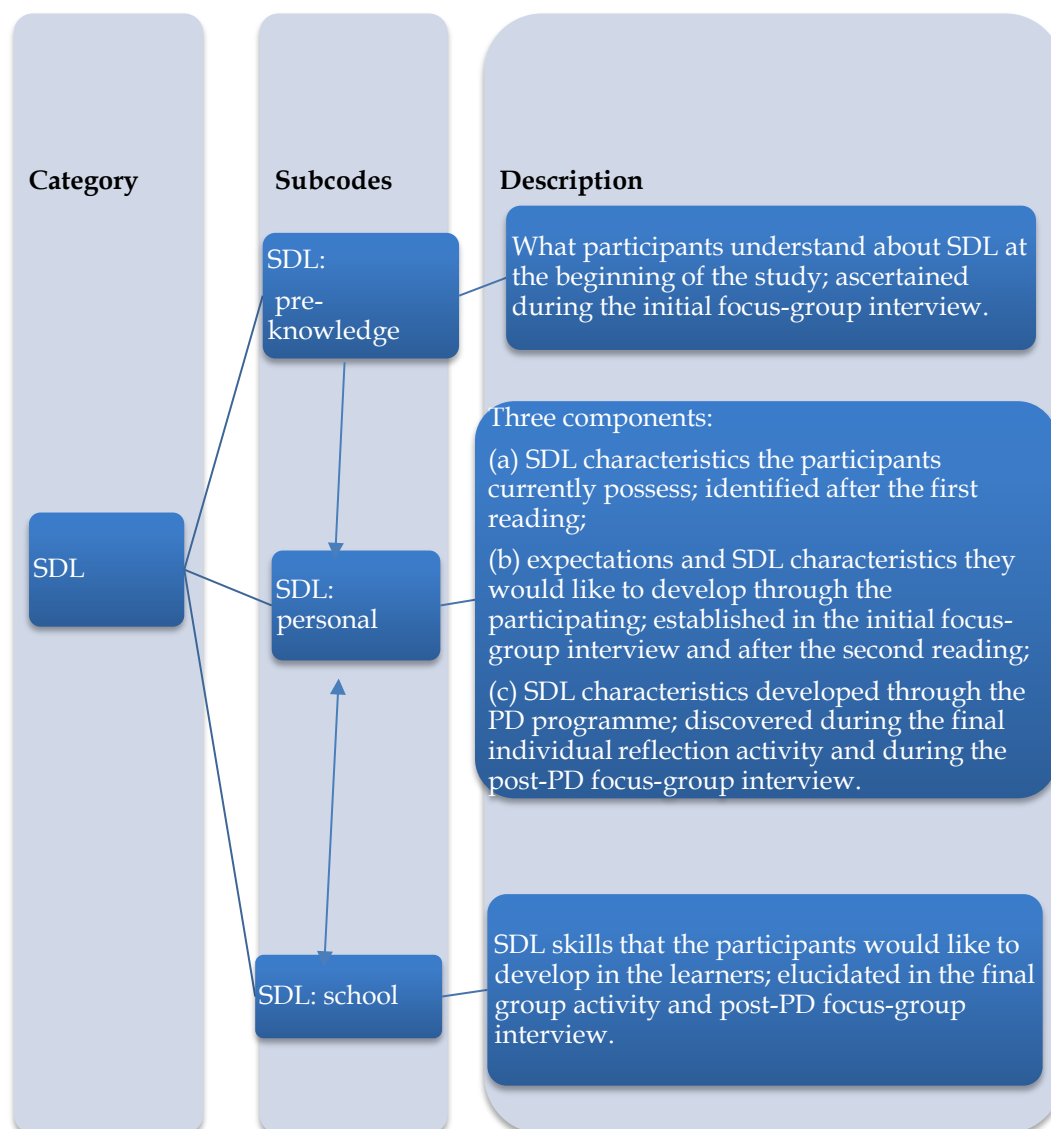


Figure 3: Descriptions of categories based on subcodes

The extended scrutinization of the data, repetitive coding and recording cycles and the audit trail that noted analytic decisions and the development of themes, were strategies employed in place of inter-rater reliability to increase analytic trustworthiness (Nowell et al. 2017). The critical inspection of interpretations that emerged were supported by reflexive memo writing and the refinement of themes was facilitated by constantly comparing data sources. Additionally, by systematically applying Braun and Clarke's (2006) six-phase framework, analytic openness and procedural consistency were ensured.

4. Ethical Considerations

According to Creswell (2009), the ethical considerations in a study are significant to ensure the researcher's protection of participants, development of a relationship of trust, promotion of the research's integrity and to prevent transgressions against the researcher's institution. Informed consent, ethical code, and participant confidentiality were observed in conducting the study. Informed

consent was obtained through an independent person who distributed consent letters outlining the research purpose, procedures, data use, risks, voluntary participation, right to withdraw and researcher contact details (Ciuk & Latusek, 2018; Fleming & Zegwaard, 2018). Ethical clearance was granted in line with North-West University's Faculty of Education ethics committee requirements, with approval from the scientific committee, signed confidentiality agreements by supervisors and legal permission from the private school's authorities. Participant confidentiality and anonymity were addressed by clarifying the limits of confidentiality in focus-group settings (Gibbs, 1997), encouraging external confidentiality and ensuring secure handling of data through the use of pseudonyms and redaction of identifiable information (Sim & Waterfield, 2019).

5. Results

The analytical narrative report of the results is presented according to three themes: self-directed learning, big ideas in mathematics and challenges in teaching and learning mathematics. Within each theme, the results unfolded according to the data-collection points: the initial focus-group interview, the BL PD programme activities and the post-PD focus-group interview. An interpretive explanation that links findings to the research questions and wider implications is included in each section.

5.1 Initial focus-group interview data

This data collection resulted from the initial focus-group interview to gain information about the participants' backgrounds, prior knowledge and expectations, before their involvement in the BL PD programme.

5.1.1 Participants' shared and unique backgrounds

In terms of gender, four females and only one male participated. The male participant was not South African and obtained his teaching degree outside of the country. The participants' racial composition included white, black and mixed-race. Their teaching experience ranged from 18 to 30 years, with varying private school mathematics experience and time employed at the research site school (Table 2). Hence, the participants' perceptions of SDL and mathematics pedagogy, may have been influenced by their diverse backgrounds and teaching experience. This interpretation highlights the need for differentiated PD approaches.

5.1.2 The big ideas in mathematics

Most participants were unfamiliar with the big ideas in mathematics, as indicated in quotes such as:

"I have no cooking clue." (P2)

"I don't have any knowledge, and I don't have any experience with the big ideas in mathematics." (P3)

"I haven't got a clue what you are talking about." (P4)

Only one participant attempted an explanation.

"...there are a couple of big ideas depending on your point of view in terms of what is important to you that you can manage those

big ideas in that direction. But I think that they're vast ideas around the subject." (P1)

None of the participants had a clear understanding of the big ideas. This confirms a knowledge gap even among experienced teachers, signifying that PD about big ideas is essential for enhancing curriculum coherence and pedagogy.

Table 2: Participants shared and unique backgrounds

Number of years of...	P1	P2	P3	P4	P5
Teaching experience	18	30	21	25	27
Mathematics teaching experience in South Africa	18	18	21	25	22
Life experience	47	53	45	51	51
Mathematics private school experience	18	18	8	22	22
Mathematics teaching experience at research site school	5	8	25	21	2
Mathematics SP phase experience	11		3	22	4
Mathematics FET phase experience	11	18	17		
Number of years of...	P1	P2	P3	P4	P5
Primary School Mathematics teaching experience					22
High School Mathematics teaching experience	11	18	21	25	

5.1.3 Challenges in teaching and learning mathematics

The responses were phase and grade specific. The primary school teacher referred to the challenge of teaching problem-solving as a skill rather than a specific topic. The senior phase teacher identified the time of the year and the grade to which topics were taught as influential. However, half of the high school teachers identified Euclidean Geometry as a challenging topic and half of them named Probability as problematic. Half of these participants linked teaching challenges to their own lack of confidence or competence in teaching the topic.

"Probability. To me it's the challenge, but I think it's my weakness. So, it becomes their weakness." (P2)

"We had quite a challenge with Euclidean geometry because it's something you see, or you can't see it...there's also probability. But I think it's because we didn't have probability the syllabus when we started as teachers. So, we had to teach ourselves and am I doing it right?" (P3)

This indicates that teacher knowledge and efficacy and curriculum structure affect the teaching of difficult topics. Hence, highlighting the areas that require targeted support.

5.1.4 Self-directed learning

All the participants could guess the meaning of SDL, but most connected SDL to improving or changing their teaching practices:

"Learn about yourself put that into your teaching. Self-directed learning is about trying things a different way." (P4)

Only one of the participants referred to motivation as an SDL quality:

"Self-directed learning is what motivates what I do in a classroom space..." (P1)

Thereby suggesting that before the BL PD programme, SDL was a means to improve teaching rather than for personal or professional growth reasons.

5.1.5 Expectations

Three out of the five participants' interests were piqued to learn more about the big ideas in mathematics and all five participants, unbeknownst to them, referred to the potential to develop an aspect of SDL particularly through collaboration, inspiring new methodologies and personal growth.

"I want to know what the big ideas one are and self-directed learning." (P4)

"So maybe I can learn something for myself from this, whether it be to find out what the big ideas or Something ideas, spend time doing something for me." (P3)

"Maybe that's, that's an opportunity maybe not inspiring also to finish doing my masters." (P2)

This suggested there was intrinsic motivation to participate and that the participants were ready to actively engage in the BL PD programme

5.2 Blended-Learning Professional-Development Programme Data

This section presents the engagement and learning of the participants during the BL PD programme focusing on mathematics big ideas, teaching and learning challenges and SDL development.

5.2.1 The big ideas in mathematics

After reading the prescribed articles for Activity 2, it was evident that the participants had acquired knowledge of the big ideas in mathematics.

"Big ideas in mathematics are those statements that are located within the principles and rules of the subject of mathematics... These interconnected sub-units define the content framing of a Mathematics curriculum." (P1)

To me these ideas are intertwined, each idea builds on the next idea forming the scaffolding to a greater understanding of why we do what we do (P2).

"...topics should not be taught in isolation as separate concepts but integrated creating a holistic theme integrating all aspects of maths." (P3)

"My understanding of the Big Ideas in Mathematics is that different 'parts' of Mathematics are shown to be connected and that these 'parts' extend across phases of the school journey." (P4)

"Big ideas make connections across a wide range of mathematical understandings." (P5)

The benefits of understanding big ideas in mathematics for teaching included providing the learners with a more coherent and integrated learning experience. Furthermore, emphasizing the interconnectedness of the different mathematical concepts across the grades provides the learners with a clearer idea of their mathematical journey. All the participants acknowledged that this approach also aids learners in observing the relevance of mathematics to real-world applications. As a researcher, this highlights the importance of examining how such understandings are enacted in classroom practice and supported through curriculum and professional learning. The collaborative Activity 3, allowed the application of horizontal and vertical curriculum continuity to a big idea in the South African CAPS, displaying practical understanding was attained.

The individual voice-note reflection Activity 4 revealed that the big ideas in mathematics are fluctuating, they can be reviewed regularly and sometimes they are not big, they can start as small to be attainable. In the SP, the big ideas in mathematics are easily identifiable broader topics in the CAPS documents. They must be taught thoroughly in the SP or gaps will exist in the FET Phase. Participants identified that teaching according to the big ideas allows for real-world relevance and for the possibility of developing problem-solving skills. It was also identified that emphasis on horizontal (within grades) and vertical (across grades) continuity promotes coherent learning progression. Two of the participants referred to the big ideas as building blocks. This sparked thinking around the teaching context and initiated a desire to implement changes to the primary school silo mentality of teaching mathematics.

The results for the final group Activity 5 are presented in Table 3.

Table 3: Results of collaborative voice-note reflection

Continue to implement	Department meetings, discussions and coordination by the head of department. Revising and reviewing teaching methods.
Suggestions for the future	Teachers must be fluid in teaching across grades. Collaborative teaching opportunities to improve the transition between primary school and high school. Implement knowledge in 2024 planning to improve transition from SP to FET. Compile a skills list for each big idea in mathematics to track progression across grades.

Therefore, the BL PD programme changed participants' understanding of big ideas from disjointed knowledge to a organized, curriculum-intergrated viewpoint, preparing them to teach mathematics more cohesively.

5.2.2 Challenges in teaching and learning mathematics

The two group activities revealed some of the challenges in teaching and learning mathematics. In Activity 3, the challenging topics chosen were Algebra (Group 1) and Probability (Group 2). The teaching challenges identified by Group 1's Activity 5 discussion were Grade 8 learners coming from other surrounding schools and not from the private school's own primary school, sometimes made worse by COVID-related syllabi interruptions. The learning challenges identified were the learners' lack of work ethic and lack of foundational knowledge.

Curriculum continuity was impeded by primary school teachers teaching the same grade and becoming stuck in their ways. The Group 2 discussion centred around a solution for teaching their identified challenging topics: teaching differently with more emphasis on real-life applications and practical activities in the lower grades to set up the more abstract nature of the topics in the higher grades. Consequently, successful mathematics instruction is impacted by both teacher and learner factors. The suggested solutions include collaborative planning and practical activity interventions in the lower grades. This indicates the BL PD programme's role in developing participants' problem-solving beyond personal skill development.

5.2.3 Self-directed learning

Four of the participants read and carried out the instructions properly for Activity 1, expressing SDL characteristics that they would like to develop. The most commonly mentioned SDL traits that the participants identified as already possessing were initiative, independence, self-discipline, time management and the ability to plan. The desired SDL traits included a strong desire to change, a tolerance of ambiguity, developing self-confidence and creativity. The group activities fostered collaboration and the traditional autonomy of SDL. The submissions displayed sharing knowledge, critically reflecting and negotiating meaning.

The SDL skills developed during the BL PD programme according to the voice-note reflections submitted for Activity 4 are displayed in Figure 4:

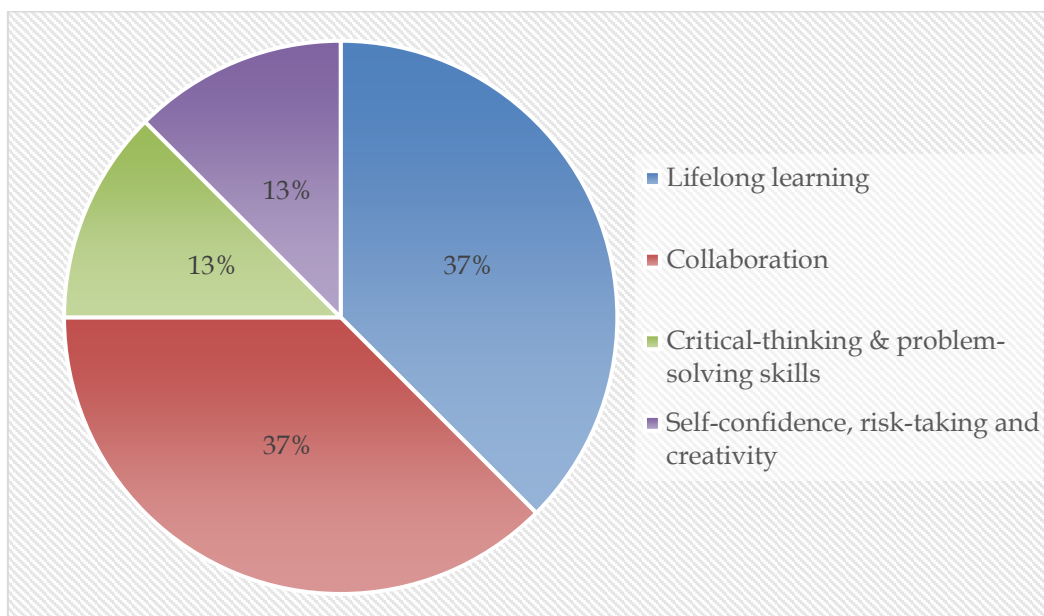


Figure 4: The SDL skills developed through the BL PD programme

Conversely, one participant revealed that some SDL characteristics, tolerance of ambiguity, collaboration and creativity, were not fully developed through the PD programme.

"The second thing that I thought needed work was a tolerance of ambiguity. Unfortunately, I am still not good at this because I need to know in black and white exactly what is wanted and how you want me to do it or to be left to do it my own way." (P4)

"Because I don't teach and play and work so well with others, I prefer to just have the creativity come from me and to serve my purpose..." (P4)

Certain SDL skills were enhanced by the BL PD programme, however, others necessitate longer-term or structured practice, indicating uneven SDL development and may depend on individual temperaments.

5.3 Post-Professional-Development Focus-Group Interview Data

The post-PD focus-group interview investigated reflections on big ideas in mathematics, the challenges faced and SDL, emphasizing personal and institutional benefits.

5.3.1 The big ideas in mathematics

The results indicated participants' reflection on present teaching practices and envisioning future improvements, specifically with respect to curriculum continuity and collaboration. These results are displayed in Table 4.

Table 4: Teaching according to the big ideas in mathematics

	Big ideas in mathematics presently	Big ideas in mathematics in the future
Curriculum continuity	The progression from primary school mathematics to high school mathematics is already partial achieved with meetings about content covered. The yearly plans are submitted to the head of the department in the high school.	Teaching according to the big ideas will further enhance curriculum continuity. Creating a formal skills list to be used across the whole school will enhance curriculum continuity especially for the significant transitions in the educational journey and save on teaching time. No reteaching, simply revising will occur.
Collaboration	The high school department meetings. The high school team teaching across all grades.	Sharing resources and ideas will further enhance collaboration and encourage interrogation of current teaching methodology.

This indicates the practical implications of big ideas for teachers including progression planning, reducing reteaching and fostering collaboration. Thus, demonstrating the implementation of the BL PD learning into teaching practice.

5.3.2 Challenges in teaching and learning mathematics

The results indicated that persistent challenges,

- that hinder teaching are time constraints, isolated topics and teachers, specifically in primary school and
- that hinder learning are COVID-related learning gaps, lack of trust between teachers and learners' work ethic.

Therefore, even though BL PD advanced teacher skills and knowledge, organizational context barriers remain, underlining that continuous PD and systemic interventions are essential.

5.3.3 Self-directed learning

There were personal benefits of SDL and SDL benefits for the school. These are recorded in Table 5.

Table 5: Personal and school SDL benefits

	Personal benefits	Benefits for the school
SDL	Initial perception changed due to BL PD programme.	The importance of developing learners' SDL was realized.
Collaboration	Participants possess various weaknesses and strengths but working together the strengths can be utilised.	A need for more teacher collaboration across the school was highlighted.
Motivation	The BL PD programme inspired content reading.	Motivation to incorporate new techniques and methods to make mathematics more enjoyable.

Table 5 demonstrates that the BL PD programme resulted in positive personal and general transformations, which indicates that SDL can reach from personal skills to wider educational practice. Collaboration between teachers and across the school was identified as key supporting factor of SDL. Motivation was also enhanced; personal motivation to engage in content reading and take greater responsibility for their learning and school motivation to employ new teaching strategies to improve learner engagement with mathematics.

6. Discussion

This study aimed to answer the primary research question: *How can teachers' self-directed learning be enhanced by developing knowledge of big ideas in mathematics through a blended learning professional development programme?* The findings' interpretation is deepened in this discussion, by explicitly correlating them within the body of scholarship, elaborating on the practical and theoretical insinuations and highlighting the novelty and importance of the study within the context of South Africa.

6.1 Big Ideas in Mathematics

It is evident from the findings that the participants had never heard about the big ideas in mathematics before attending the initial focus-group interview. This confirmed the researcher's suspicion expressed in the introduction, that some South African mathematics teachers are not privy to the big ideas in mathematics. In contrast with the international literature, where curriculum coherence and the design of instruction are informed by big ideas (Charles, 2005; Siemon, 2022). This elucidates a contextual difference between global and local professional knowledge.

The findings of the BL PD programme activities, specifically Activities 2- 5, showed that the participants attained a better understanding of the big ideas in mathematics and the benefits of knowing and teaching according to the big ideas in mathematics. However, the Activity 3 submissions displayed progressively deeper understanding when the big ideas were collaboratively applied to the CAPS documents. This mirrors Siemon's (2022) claim that teachers who know about the big ideas in mathematics and their interconnections, understand the learning progression more proficiently. By the time that the reflection Activities 4 and 5 were completed, the emerging plans to improve transitions through grades and phases, to relook at lesson approaches and to design big ideas skills, demonstrate transformation from theoretical comprehension to practical application.

A prominent disparity was evident between the primary school and high school contexts. Although the high school participants were proud to from working with these "building blocks" and established collaborative teaching practices, the one primary school participant was at a disadvantage, revealing their experience of the confining effects of siloed instructional practices. This finding slightly refines the existing literature by demonstrating how the school context can limit teachers' knowledge of, and experience with, the big ideas in mathematics.

6.1.1 Professional Development and Blended Learning

Bayar's (2014) criteria for impactful PD: relevance to personal and institutional needs, active involvement, long-term engagement and quality facilitators mostly aligned with the success of the BL PD programme. The BL model allowed the participants convenient pacing and location, which enabled sustained engagement, despite time constraints – it was positioned nearing the high school's examination period. The BL PD programme promoted reflection, discourse and iterative learning, that aided more in-depth professional growth, unlike once-off traditional workshops.

Contrary to Bayar's (2014) criteria, the programme was short-term. However, participants made mention of their intentions to explore the concept of the big ideas in mathematics further, thus suggesting the initiation of long-term PD opportunities. This finding corroborates and extends existing PD literature by displaying that provided when a BL PD programme is aligned with teachers' immediate professional needs, even when offered for a limited time, they can act as a catalyst for ongoing SDL.

6.2 Self-Directed Learning

In the initial focus-group interview, the participants were explicitly asked what they knew about SDL. The findings implied that the participants' knowledge was limited and when they tried to explain this limited understanding, most of the explanations referred to improving or changing their teaching. However, due to the word "self" in the terminology, they recognized that it was about them personally. This finding aligns with an earlier study by Loeng (2020) that suggests that even without conceptual understanding, they engage in SDL implicitly.

After completing Activity 1 of the BL PD programme, it was evident that the participants had acquired a better understanding of SDL. They were able to identify the SDL characteristics they already possess and flag the SDL skills that they wanted to develop. Consistent with Johnson and Johnson (2019), Oswald (2003) and Sebatana and Dudu (2021), collaboration emerged as a salient aspect of SDL. The findings indicated that the structured collaborative tasks and the focus-group interviews fostered collaboration and initiated and sustained discussions, respectively. Although Activities 3 and 5 did not deal directly with SDL, the participants' responses and completion of the activities indicated that the collaborative and reflexive nature of the activities did encourage SDL.

Notably, the collaborative completion of Activity 3 and the redistribution of tasks in Group 1, when one of the participants was ill, displayed how SDL enacted within a community of practice, extending SDL beyond individual autonomy to group agency. This displayed the participants' sharing of knowledge, resources and support for each other. It was evident from the recorded dialogue submissions for Activity 5, that the participants were able to critically reflect on, and negotiate meaning about, what they had learnt and were able to apply their knowledge to their current working context; Group 1 discussed the creation of a skills list to be used across the school in mathematics and improving the transition between primary school and high school. Group 2 discussed teaching methods for

challenging topics in SP; rather focusing on practical work to form a foundation to build on the abstract explanations in the FET Phase.

Finally, in the discussion of the findings of the post-PD focus-group interview, it was revealed that the participants' perception of SDL had developed since the initial focus-group interview and they attributed this change in perception to engagement in the BL PD programme. The findings indicated that the participants have realized the value of teamwork. The participants working in the high school appreciated the benefit of drawing on each other's strengths contrary to the one participant working in the primary school who identified the need for more collaboration and fluidity of mathematics teachers and less isolation and rigidity in constantly teaching only one grade.

A strong theoretical lens for interpreting these findings is provided by Garrison's (1997) Comprehensive Model of SDL and the findings aligned with the literature highlighting the key aspect of SDL in learners managing and taking charge of their own process of learning (Abdullah, 2001; Loeng, 2020). The BL PD programme enabled transactional control through flexible pacing, choice of learning environments, and discursive assistance from the researcher. Four out of the five activities in the BL PD programme involved reflection and critical thinking which are related to learners' metacognitive proficiency. This enabled participants to demonstrate self-management through independent planning, self-monitoring through the reflective activities and feedback from the researcher. Their motivation was sustained and evidenced by full participation and completion of all BL PD activities.

This study demonstrated that a meticulously designed BL programme can support SDL without reducing the learner autonomy as opposed to previous studies that frequently report that with teacher PD sustaining SDL is a challenge. This finding extended Garrison's model by elucidating how BL programmes can make SDL principles actionable in teacher PD. There is evidence in the findings that the private school's mathematics teachers' perceptions of SDL were altered from the beginning of the study to the end of the study. The findings suggested that the participants were planning to further enhance their own SDL and even aimed to develop those of the learners that they teach.

6.3 Theoretical Implications

The theoretical contribution of the study is that it strengthens the conceptual connection between mathematics education, SDL and BL. The findings align with the literature, that personal competence and group competence can be affected by PD (Zambo & Zambo, 2008), indicating that participants have identified their own strengths and weaknesses through the BL PD programme and have recognised, particularly in the high school, that teamwork results in different strengths of the members in the department being utilised for the benefit of the learners. This demonstrates that SDL is not merely about individual endeavours but can be enhanced through collaborative BL programmes. Moreover, integrating big ideas within a BL PD programme demonstrates how subject-specific knowledge acts as a catalyst for SDL enhancement. This strengthens existing SDL theory by

highlighting the function of subject coherence and instructional purpose in upholding learners' engagement and motivation.

6.4 Practical Implications for South African Teacher Professional Development

The practical implications of the findings suggest the intentional integration of the big ideas in mathematics in South African PD as guiding frameworks for planning the curriculum and teacher collaboration. The direct implications of improving curriculum continuity across grades and phases will be realised as a result of a big-ideas-based skills list. Furthermore, the study underscores the necessity for PD programmes that diminish primary school teacher isolation and encourage cross-grade teaching and collaborative planning. The BL PD programme's success further implies that existing digital platforms, such as Google Platform, can be leveraged to provide PD that is cost-effective and able to reach many teachers efficiently.

6.5 Novelty and Significance of the Study

The study's novelty is its integration of SDL, BL and big ideas in mathematics within the PD context of South Africa – this combination has barely been explored in existing research. The study offers a meaningful contribution to mathematics education scholarship by demonstrating how a BL PD programme can concurrently enhance teachers' SDL and extend subject-specific pedagogical understanding. Its ability to inform scalable PD programmes intended to improve curriculum coherence and collaboration amongst teachers in South African schools, further reiterates its significance.

6.6 Contributions of the Study

The contribution of this study to the body of scholarship is that it emphasizes the significance of enhancing SDL in experienced private school mathematics teachers and that a BL PD programme has the potential to achieve this objective. The results suggested that the big ideas in mathematics are worthwhile content to include in a mathematics PD programme. Moreover, this study highlighted the importance of teachers acquiring knowledge of the big ideas in mathematics. Especially, towards curriculum continuity and presenting a more interconnected perspective of mathematics to learners. These findings have the potential to close the gap identified in the literature and to form the basis for designing and creating in-service PD programmes and possibly even pre-service training programmes for mathematics teachers in the future.

7. Conclusion

This study researched how a private school's mathematics teachers' SDL could be enhanced through the development of knowledge of the big ideas in mathematics, within a BL PD programme. The findings indicate that the BL PD programme was most conducive for the mathematics teaching profession, offering flexibility, accessibility and collaboration which encourages meaningful professional learning. Drawing on insights from the body of literature, the researcher utilized suggestions to create an effective BL PD programme that integrated principles of SDL with subject-specific mathematical knowledge. The empirical evidence indicates that participating teachers strengthened key SDL characteristics – such

as reflection, collaboration, self-monitoring and motivation - whilst simultaneously acquiring and deepening their knowledge of the big ideas in mathematics. Significantly, the study demonstrates that when PD is organized around both SDL and curriculum coherence, teachers are better positioned to take responsibility for their PD and to practically use new knowledge to enhance curriculum continuity and their teaching practice.

Further research should be carried out to include this type of programme in pre-service mathematics teacher training. This could possibly initiate SDL characteristics from the start of teachers' careers and the knowledge of big ideas in mathematics could potentially influence their view of mathematics as a connected whole. Likewise, it could impact on the way that they teach learners; to see mathematics as a series of interconnected topics rather than isolated content. Alternatively, in future studies a quantitative component could be included where participants' perceptions of their own SDL can be measured through a quantitative questionnaire such as the Self-Directed Learning Instrument that assesses the four SDL areas; motivation of learning, planning and implementation, self-monitoring and relational communication (Cheng et al., 2010).

8. Limitations of the study

Although the study generated worthy insights, several limitations must be acknowledged. The population was confined to one private school at which the researcher is an employee. Another private school may not be familiar with the Google Platform to partake in the BL PD programme. This and the fact that the participants may not have been as comfortable talking about themselves with the researcher may hinder the success of the PD programme. The researcher was dependent on the participants to become fully engaged in the BL PD programme. The time of the year when the BL PD programme was conducted limited the participants to fully invest in themselves.

The number of SP and FET mathematics teachers may limit the success of a similar BL PD programme. Fewer teachers would mean less collaboration and too many teachers would lose the intimate nature of the focus-group interviews. Another limitation observed was that primary school and high school teachers added different perspectives and with only one primary school teacher participating in the study, this possibly created a warped sense of how SDL and the big ideas in mathematics could be utilised at the primary school level. Finally, public school teachers may have limited access to wi-fi and devices and therefore it would not be feasible to conduct a BL PD programme. This would negate the development of some of the SDL characteristics in terms of control and the online collaboration that proved to be beneficial in this study.

9. Recommendations

A recommendation would be to conduct the BL PD programme at several private schools simultaneously. Thus, ascertaining whether the participants would be comfortable to engage openly about their SDL characteristics and possible lack of knowledge of the big ideas in mathematics, without personally knowing the

researcher. Another possibility would be to extend the study to include the Foundation Phase (FP) and Intermediate Phase (IP) mathematics teachers at private schools. This could determine whether primary school mathematics teachers typically teach a single grade over extended periods or whether primary schools—similar to the high school context in this study—are adopting more fluid teaching practices that focus on the big ideas in mathematics and promote curriculum coherence.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

The study has not received any funding.

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